8.3 Further Complex Numbers

Question Paper

Course	CIE A Level Maths
Section	8. Complex Numbers
Topic	8.3 Further Complex Numbers
Difficulty	Very Hard

Time allowed: 40

Score: /29

Percentage: /100

Question 1

Express the following complex numbers in exponential form:

(i) $-5(\cos 2 - \mathrm{i}\sin 2)$

(ii) $\left(\sqrt{2}-\sqrt{6}\right)-\left(\sqrt{2}+\sqrt{6}\right)\mathrm{i}$

[4 marks]

Question 2

$z_1 = 14\mathrm{e}^{9\mathrm{i}}$

$z_2 = 10\mathrm{e}^{-2\mathrm{i}}$

(i) Work out $z_1 z_2$ and $\frac{z_2}{z_1}$, giving your answers in exponential form.

(ii) Express your answers to part (i) as complex numbers in modulus-argument form. In each case the modulus and argument should be given as exact values, with the argument θ being given in the interval $-\pi < \theta \leq \pi$.

[4 marks]

Question 3a

Given the points 1 and z on an Argand diagram, where $z \neq 0$ is a complex number, explain how to find each of the following points by geometrical construction. In each case provide a sketch to illustrate your answer.

(a) z^2

[3 marks]

Question 3b

(b) $(2 - \mathrm{i})z$

[3 marks]

Question 4a

Let $z = re^{i\theta}$ be a general complex number, where $r, \theta \in \mathbb{R}$ and $r \geq 0$.

(a) Use the geometry of complex numbers to explain why

$$re^{i\theta} = re^{i(\theta+2\pi)}$$

for any value of θ.

[2 marks]

Question 4b

(b) Hence use the properties of complex numbers to determine the two distinct square roots of z, giving your answers in exponential form in terms of r and θ.

[3 marks]

Question 5

$$z = \sqrt{3} - i, \quad \text{Im}\left(\frac{z^2}{w}\right) = 0, \quad \left|\frac{z^2}{w}\right| = \frac{1}{2}|z|$$

Use geometrical reasoning to find the two possibilities for w, giving your answers in exponential form.

[4 marks]

Question 6a

Note: You may assume throughout this question that $\mathrm{i} = \sqrt{-1}$ behaves exactly the same as any other constant for purposes of algebraic manipulation, differentiation and integration.

(a) For a complex number $z = \cos\theta + \mathrm{i}\sin\theta$, where $\theta \in \mathbb{R}$, show that

$$\frac{\mathrm{d}z}{\mathrm{d}\theta} = \mathrm{i}z$$

[2 marks]

Question 6b

(b) Utilising your knowledge of differential equations, explain briefly why the result of part (a) supports the validity of *Euler's relation*

$$\mathrm{e}^{\mathrm{i}\theta} = \cos\theta + \mathrm{i}\sin\theta$$

[4 marks]